1. Transient Plane Wave

Here we discuss the specification of an incident plane wave using the coordinates shown in Figure 1, where we denote the unit vectors of the spherical coordinate system as $\hat{\boldsymbol{a}}_r$, $\hat{\boldsymbol{a}}_\theta$ and $\hat{\boldsymbol{a}}_\phi$ and those of the Cartesian coordinate system as $\hat{\boldsymbol{a}}_x$, $\hat{\boldsymbol{a}}_y$ and $\hat{\boldsymbol{a}}_z$. The direction of propagation is chosen to be the $-\hat{\boldsymbol{a}}_r$ direction which is specified by the angles θ and ϕ. The polarization of the electric field vector $\boldsymbol{E}^i$ is specified by the angle ζ being measured from $\hat{\boldsymbol{a}}_\theta$ towards $\hat{\boldsymbol{a}}_\phi$, and is denoted by the unit vector $\hat{\boldsymbol{e}}$. Assuming free space, we have μ_o and ε_o, and the plane wave propagates with a speed of $v_o = (\mu_o \varepsilon_o)^{-1/2}$.

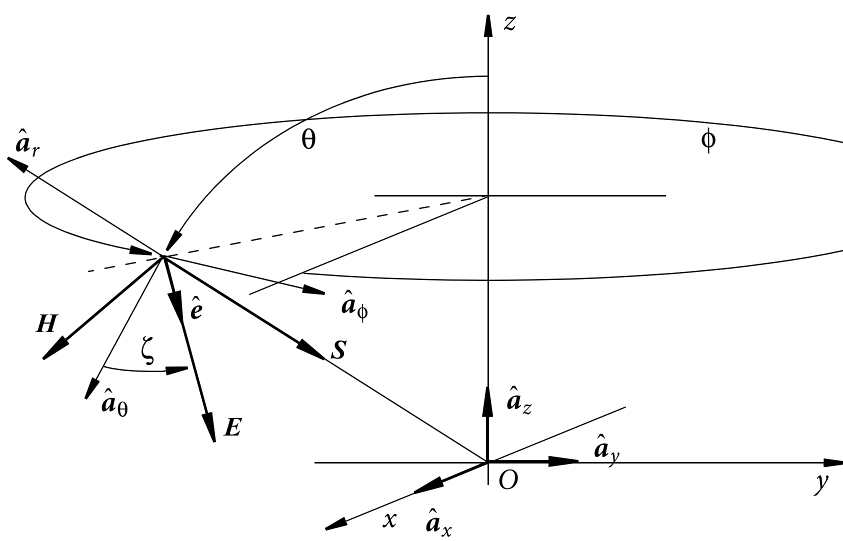

Figure 1 Specification of an incident electromagnetic wave.

Using this notation, the incident plane wave can be specified as

$$\boldsymbol{E}^i(t, \boldsymbol{r}) = \hat{\boldsymbol{e}} E_o(t - \boldsymbol{v} \cdot \boldsymbol{r} / |\boldsymbol{v}|^2) \tag{1}$$

where $\boldsymbol{r} = x\hat{\boldsymbol{a}}_x + y\hat{\boldsymbol{a}}_y + z\hat{\boldsymbol{a}}_z$ is the position vector, t is the time, and $E_o(\cdot)$ is the electric field waveform function. The polarization unit vector $\hat{\boldsymbol{e}}$ can be written as

$$\hat{\boldsymbol{e}} = e_x\hat{\boldsymbol{a}}_x + e_y\hat{\boldsymbol{a}}_y + e_z\hat{\boldsymbol{a}}_z = \cos(\zeta)\hat{\boldsymbol{a}}_\theta + \sin(\zeta)\hat{\boldsymbol{a}}_\phi = \begin{bmatrix}0 & \cos\zeta & \sin\zeta\end{bmatrix} \mathrm{R}_{sph} \begin{bmatrix}\hat{\boldsymbol{a}}_x & \hat{\boldsymbol{a}}_y & \hat{\boldsymbol{a}}_z\end{bmatrix}^T, \tag{2}$$

where R_{sph} denotes the transformation matrix between the spherical and Cartesian coordinate systems:

$$\mathrm{R}_{sph} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} . \tag{3}$$

Thus, in Cartesian coordinates, the direction of the incident electric field can be written as

$$\hat{\boldsymbol{e}} = \begin{bmatrix}\cos\theta\cos\phi\cos\zeta - \sin\phi\sin\zeta & \cos\theta\sin\phi\cos\zeta + \cos\phi\sin\zeta & -\sin\theta\cos\zeta\end{bmatrix} \begin{bmatrix}\hat{\boldsymbol{a}}_x & \hat{\boldsymbol{a}}_y & \hat{\boldsymbol{a}}_z\end{bmatrix}^T . \tag{4}$$

The velocity vector of the wave can also be written in Cartesian coordinates as

$$\boldsymbol{v} = v_x\hat{\boldsymbol{a}}_x + v_y\hat{\boldsymbol{a}}_y + v_z\hat{\boldsymbol{a}}_z = -v_o\hat{\boldsymbol{a}}_r = \begin{bmatrix}-v_o & 0 & 0\end{bmatrix} R_{sph} \begin{bmatrix}\hat{\boldsymbol{a}}_x & \hat{\boldsymbol{a}}_y & \hat{\boldsymbol{a}}_z\end{bmatrix}^T, \tag{5}$$

that is

$$\boldsymbol{v} = \begin{bmatrix}-v_o\sin\theta\cos\phi & -v_o\sin\theta\sin\phi & -v_o\cos\theta\end{bmatrix} \begin{bmatrix}\hat{\boldsymbol{a}}_x & \hat{\boldsymbol{a}}_y & \hat{\boldsymbol{a}}_z\end{bmatrix}^T . \tag{6}$$